Chapter 10.1 Sequences and Summation Notation

→ Definition of a Sequence
- Infinite sequence $\{a_n\}$ goes on forever
- Finite sequence, stops.

① Find particular terms of a sequence from the general term.

Ex) Write the first 4 terms of the sequence.

a.) $a_n = 2n+5$

$a_1 = 2(1)+5 = 7$
$a_2 = 2(2)+5 = 9$
$a_3 = 2(3)+5 = 11$
$a_4 = 2(4)+5 = 13$

$\therefore 7, 9, 11, 13$

b.) $a_n = \frac{(-1)^n}{2^n+1}$

$a_1 = \frac{(-1)^1}{2^1+1} = \frac{-1}{2+1} = -\frac{1}{3}$

$a_2 = \frac{(-1)^2}{2^2+1} = \frac{1}{4+1} = \frac{1}{5}$

$a_3 = \frac{(-1)^3}{2^3+1} = \frac{-1}{8+1} = -\frac{1}{9}$

$a_4 = \frac{(-1)^4}{2^4+1} = \frac{1}{16+1} = \frac{1}{17}$

$\therefore -\frac{1}{3}, \frac{1}{5}, -\frac{1}{9}, \frac{1}{17}$

② Use recursion formulas

Ex| Find the first 4 terms of the sequence in which $a_1 = 3$ and $a_n = 2a_{n-1}+5$ for $n \geq 2$

$a_1 = 3$
$a_2 = 2a_{2-1}+5 = 2a_1+5 = 2(3)+5 = 6+5 = 11$
$a_3 = 2a_{3-1}+5 = 2a_2+5 = 2(11)+5 = 22+5 = 27$
$a_4 = 2a_{4-1}+5 = 2a_3+5 = 2(27)+5 = 54+5 = 59$

$\therefore 3, 11, 27, 59$

Use factorial notation

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Ex) write the first 4 terms

$$a_n = \frac{20}{(n+1)!}$$

$$a_1 = \frac{20}{(1+1)!} = \frac{20}{2!} = \frac{20}{2\cdot 1} = \frac{20}{2} = 10$$

$$a_2 = \frac{20}{(2+1)!} = \frac{20}{3!} = \frac{20}{3\cdot 2\cdot 1} = \frac{20}{6} = \frac{10}{3}$$

$$a_3 = \frac{20}{(3+1)!} = \frac{20}{4!} = \frac{20}{4\cdot 3\cdot 2\cdot 1} = \frac{20}{24} = \frac{5}{6}$$

$\therefore 10, \frac{10}{3}, \frac{5}{6}, \frac{1}{6}$

$$a_4 = \frac{20}{(4+1)!} = \frac{20}{5!} = \frac{20}{5\cdot 4\cdot 3\cdot 2\cdot 1} = \frac{20}{120} = \frac{1}{6}$$

Ex) Evaluate each factorial expression

a.) $\frac{14!}{2!12!} = \frac{14\cdot 13\cdot \cancel{12!}}{2\cdot 1\cdot \cancel{12!}}$

$= \frac{14\cdot 13}{2} = \boxed{91}$

b.) $\frac{n!}{(n-1)!} = \frac{n\cancel{(n-1)!}}{\cancel{(n-1)!}} = \boxed{n}$

4 Use summation notation

Ex) Expand and evaluate the sum!

a.) $\sum_{i=1}^{6} 2i^2 = 2(1)^2 + 2(2)^2 + 2(3)^2 + 2(4)^2 + 2(5)^2 + 2(6)^2$

$= 2 + 8 + 16 + 32 + 50 + 72$

$= 182$

b.) $\sum_{k=3}^{5} (2^k - 3) = 2^3 - 3 + 2^4 - 3 + 2^5 - 3$

$= (8-3) + (16-3) + (32-3)$

$= 5 + 13 + 29$

$= 47$

c.) $\sum_{i=1}^{5} 4 = 4 + 4 + 4 + 4 + \cancel{5} 4 = 20$

$\boxed{Ex}$ Express each sum using summation notation

a.) $1^2 + 2^2 + 3^2 + \ldots + 9^2$

$\sum_{i=1}^{9} i^2$

b.) $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots + \frac{1}{2^{n-1}}$

$\sum_{i=1}^{n} \frac{1}{2^{i-1}}$

Note

Properties of Sums

1.) $\sum_{i=1}^{n} c a_i = c \sum_{i=1}^{n} a_i$, c any real #

2.) $\sum_{i=1}^{n} (a_i + b_i) = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i$

3.) $\sum_{i=1}^{n} (a_i - b_i) = \sum_{i=1}^{n} a_i - \sum_{i=1}^{n} b_i$

Chapter 10.2 Arithmetic Sequences

① Find the common difference for an arithmetic sequence.

Ex | Arithmetic sequence: $1, 3, 5, 7, \ldots$ Common differenc: $d = 3 - 1 = 2$

② Write terms of an arithmetic sequence

Ex | Write the first 6 terms of the arithmetic sequence in which $a_1 = 100$ and $a_n = a_{n-1} - 30$

$a_1 = 100$

$a_2 = a_{2-1} - 30 = a_1 - 30 = 100 - 30 = 70$

$a_3 = a_{3-1} - 30 = a_2 - 30 = 70 - 30 = 40$

$a_4 = a_{4-1} - 30 = a_3 - 30 = 40 - 30 = 10$

$a_5 = a_{5-1} - 30 = a_4 - 30 = 10 - 30 = -20$

$a_6 = a_{6-1} - 30 = a_5 - 30 = -20 - 30 = -50$

$\therefore 100, 70, 40, 10, -20, -50$

③ Use the formula for the general term of an arithmetic sequence. $a_n = a_1 + (n-1)d$

Ex | Find the 9^{th} term of the arithmetic sequence whose 1^{st} term is 6 and whose common difference is -5

$6, 1, -4, -9, -14, -19, -24, -29, \boxed{-34}$ or

$a_n = a_1 + (n-1)d$

$a_9 = 6 + (9-1)(-5)$

$a_9 = 6 + (8)(-5)$

$a_9 = 6 - 40 = -34$

] Thanks to drive-thrus and curbside delivery, Americans are eating more meals behind the wheel. In 2004, we averaged 32 a' la car meals, increasing by approximately 0.7 meal per year.

a.) write a formula for the n^{th} term of the arithmetic sequence that models the average # of car meals n years after 2003,

(b.) How many car meals will Americans average by the year 2014?

a.) $a_n = a_1 + (n-1)d$

$= 32 + (n-1)0.7$

$= 32 + 0.7n - 0.7$

$= 0.7n + 31.3$

b.) $a_{11} = 0.7(11) + 31.3$ $n = 2014 - 2003 = 11$

$= 7.7 + 31.3$

$\boxed{a_{11} = 39}$

(4) Use the formula for the sum of the first n terms of an arithmetic sequence

$$S_n = \frac{n}{2}(a_1 + a_n)$$

Ex] Find the sum of the first 15 terms...

3, 6, 9, 12, ...

$S_n = \frac{n}{2}(a_1 + a_n)$

$S_{15} = \frac{15}{2}(3 + 45)$

$S_{15} = \frac{15}{2}(48) = \boxed{360}$

to find a_{15} use general

$a_n = a_1 + (n-1)d$

$a_{15} = 3 + (15-1)3$

$a_{15} = 3 + (14)3$

$a_{15} = 3 + 42 = 45$

Ex] Find the following sum: $\sum_{i=1}^{30} (6i - 11)$

$a_1 = 6(1) - 11 = -5$

$a_{30} = 6(30) - 11 = 169$

$S_{30} = \frac{30}{2}(-5 + 169)$

$S_{30} = 2460$

Chapter 10.3 Geometric Sequences & Series

① Find the common ratio of a geometric sequence.

↳ The amount change from one number to the next.

② Write terms of a geometric sequence

Ex| Write the first 6 terms of the geometric sequence with first term 12 and common ratio $\frac{1}{2}$

$12, 6, 3, \frac{3}{2}, \frac{3}{4}, \frac{3}{8}$

③ Use the formula for the general term of a geometric sequence.

General Geometric Sequence $a_n = a_1 \overset{\text{ratio}}{r}^{\,n-1}$

Ex| Find the seventh term of the geometric sequence whose first term is 5 and whose common ration is -3.

$a_7 = a_1 r^{7-1} = a_1 r^6$

$a_7 = 5(-3)^6 = 5(729) = 3645$

$\boxed{\therefore 7^t \text{ term} = 3645}$

3)-Write the general term for the geometric Sequence 3, 6, 12, 24, 48, ...

- Then use the formula for the general term to find the 8th term.

$r = \frac{6}{3} = 2 \ , \ a_1 = 3$

$a_n = 3(2)^{n-1}$ ← general

$a_8 = 3(2)^{8-1} = 3(2)^7 = 3(128) = \boxed{384}$

4) Use the formula for the sum of the first n terms of a geometric sequence.

$$S_n = \frac{a_1(1-r^n)}{1-r} \ ; \ r \neq 1$$

Ex) Find the sum of the first nine terms of the geometric sequence 2, -6, 18, -54

$r = \frac{-6}{2} = -3 \ , \ a_1 = 2$

$$S_9 = \frac{2(1-(-3)^9)}{1-(-3)} = \frac{2(1-(-19683))}{4} = \frac{2(19684)}{4} =$$

$$\boxed{S_9 = 9842}$$

ind the following sum: $\sum_{i=1}^{8} 2 \cdot 3^i$

$a_1 = 2 \cdot 3^1 = 6$ $\quad r = \frac{18}{6} = 3$

$a_2 = 2 \cdot 3^2 = 18$

$S_8 = \frac{6(1-3^8)}{1-3} = \frac{6(-6560)}{-2} = \boxed{19680}$

Ex A job pays a salary of \$30,000 the first year. During the next 29 yrs, the salary increases by 6% each year. What is the total lifetime salary over the 30-year period?

$a_1 = 30000$

$r = 1.06$

$S_{30} = \frac{30000(1-1.06^{30})}{1-1.06} = \frac{30000(-4.74)}{-0.06}$

$\boxed{S_{30} = \$2,371,746}$

Find the value of an annuity

Value of an Annuity $A = \dfrac{P\left[\left(1+\frac{r}{n}\right)^{nt}-1\right]}{\frac{r}{n}}$

Ex) At age 30, to save for retirement, you decide to deposit \$100 at the end of each month into an IRA that pays 9.5% compounded monthly

a.) How much will you have from the IRA when you retire at age 65?

b.) Find the interest

$P=100 \quad r=0.095 \quad n=12 \quad t=35$

a.) $A = \dfrac{100\left[\left(1+\frac{0.095}{12}\right)^{12\cdot 35}-1\right]}{\frac{0.095}{12}}$

$A = \$333,946$

b.) Interest = Value of IRA − Total deposits

$= 333946 - 100 \cdot 12 \cdot 35$

$= 333946 - 42000$

$\boxed{= \$291,946}$

Use the formula for the sum of an infinite geometric series.

$$S = \frac{a_1}{1-r}$$

Ex| Find the sum of the infinite geometric series:

$$3 + 2 + \frac{4}{3} + \frac{8}{9} + \ldots$$

$$a_1 = 3 \quad r = \frac{2}{3}$$

$$S = \frac{3}{1-\frac{2}{3}} = \frac{3}{\frac{1}{3}} = 9$$

∴ The sum of this infite geometric series is 9.

Ex| Express $0.\overline{9}$ as a fraction in lowest terms.

$$0.\overline{9} = 0.9999\ldots = \frac{9}{10} + \frac{9}{100} + \frac{9}{1000} + \ldots$$

$$a_1 = \frac{9}{10} \quad r = \frac{\frac{9}{100}}{\frac{9}{10}} = \frac{1}{10}$$

$$S = \frac{\frac{9}{10}}{1-\frac{1}{10}} = \frac{\frac{9}{10}}{\frac{9}{10}} = 1$$

∴ An equivalent fraction for $0.\overline{9}$ is 1

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Chapter 10.5 The Binomial Theorem

① Evaluate a binomial coefficient

n above r $-\binom{n}{r} = \frac{n!}{r!(n-r)!}$

${}_nC_r$ used in place of $\binom{n}{r}$

Ex) Evaluate:

a.) $\binom{6}{3} = \frac{6!}{3!(6-3)!}$

$= \frac{6!}{3!3!} = \frac{6\cdot5\cdot4\cdot\cancel{3!}}{3\cdot2\cdot1\cdot\cancel{3!}} = \frac{120}{6} = \boxed{20}$

b.) $\binom{6}{0} = \frac{6!}{0!(6-0)!} = \frac{\cancel{6!}}{0!\cancel{6!}} = \boxed{1}$

c.) $\binom{8}{2} = \frac{8!}{2!(8-2)!} = \frac{8!}{2!6!}$

$= \frac{8\cdot7\cdot\cancel{6!}}{2!\cancel{6!}} = \frac{56}{2} = \boxed{28}$

d.) $\binom{3}{3} = \frac{3!}{3!(3-3)!} = \boxed{1}$

② Expand a binomial raised to a power

$$(a+b)^n = \binom{n}{0}a^n b^0 + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \binom{n}{3}a^{n-3}b^3 + \dots + \binom{n}{n}b^n$$

$$= \sum_{r=0}^{n}\binom{n}{r}a^{n-r}b^r$$

Ex) Expand $(x+1)^4$

$$(x+1)^4 = \binom{4}{0}x^4 + \binom{4}{1}x^{4-1}1 + \binom{4}{2}x^{4-2}1^2 + \binom{4}{3}x^{4-3}\cdot1^3 + \binom{4}{4}1^4$$

$$= \frac{4!}{0!4!}x^4 + \frac{4!}{1!3!}x^3 + \frac{4!}{2!2!}x^2 + \frac{4!}{3!1!}x + \frac{4!}{4!0!}\cdot1$$

$$= x^4 + 4x^3 + 6x^2 + 4x + 1$$

Ex] Expand $(x-2y)^5$ **negatives**

$(x-2y)^5 = \binom{5}{0}x^5 + \binom{5}{1}x^4\cdot 2y + \binom{5}{2}x^3\cdot(2y)^2 + \binom{5}{3}x^2(2y)^3 + \binom{5}{4}x^1(2y)^4 + \binom{5}{5}(2y)^5$

$= \frac{5!}{0!5!}x^5 + \frac{5!}{1!4!}x^4\cdot 2y + \frac{5!}{2!3!}x^3\cdot 4y^2 + \frac{5!}{3!2!}x^2\cdot 8y^3 + \frac{5!}{4!1!}x\,16y^4 + \frac{5!}{5!0!}32y^5$

$= x^5 + 10x^4y + (10)4x^3y^2 + (10)8x^2y^3 + 5x\cdot 16y^4 + 32y^5$

$= x^5 + 10x^4y + 40x^3y^2 + 80x^2y^3 + 80xy^4 + 32y^5$

③ Find a particular term in a binomial expansion,

$$\binom{n}{r}a^{n-r}b^r$$

Ex] Find the 5^{th} term in the expansion of $(2x+y)^9$

fifth term $= \binom{9}{4}(2x)^{9-4}\cdot y^4$

$= \binom{9}{4}(2x)^5y^4$

$= \frac{9!}{4!5!}\cdot 32x^5\cdot y^4$

$= \frac{9\cdot 8\cdot 7\cdot 6\cdot \cancel{5!}}{4!\,\cancel{5!}}\cdot 32x^5y^4$

$= 126\cdot 32x^5y^4$

$= \boxed{4032x^5y^4}$